On the Parameter Estimation for Compartment Models with Uncertain Data

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Consider a mammillary model with one central compartment communicating with $p-1$ peripheral compartments. The solution of the homogeneous system of the governing equations is given by

$$f(x,t) = \sum_{j=1}^{p} x_{2j-1} \exp(-x_{2j}t), \quad x \in \mathbb{R}^n, \quad n = 2p.$$

The goal is to estimate the **set** of all parameter values (amplitudes x_{2j-1} and decay constants x_{2j}, $j = 1, \ldots, p$), which are consistent with the observations $\tilde{y}_i$ contained in given intervals $[\underline{y}_i, \overline{y}_i]$, that is $\underline{y}_i \le f(x, t_i) \le \overline{y}_i$, $i = 1, \ldots, m$. The methodology for guaranteed parameter set estimation of exponential sums, proposed in [1], is based on Prony's method for interval-valued data. Since exponential sums may be very sensitive to changes in their parameters, special emphasis is put on finding sharp bounds for the zero set of an interval polynomial and for the solution set of two parametric interval linear systems with particular structure of the parameter dependencies (Hankel- and Vandermonde-type).

This work is focused on the parametric interval linear systems with Hankel-type dependencies. In [1] the parametric solution set is bounded by Bernstein expansion of the symbolic solution components. The method we propose requires range enclosure of rational functions involving k interval parameters, where k can be chosen as $k = 1$ or any other number $1 \le k \le n-1$, n is the dimension of the system. Our method can be used either as an alternative to the method based on Bernstein expansion applied in [1], or as a preprocessing step reducing the number of the parameters for the Bernstein expansion. This allows handling linear systems with larger number of parameters and large parameter intervals.

References

[1] J. Garloff, I. Idriss, A.P. Smith, *Guaranteed parameter set estimation for exponential sums: three-terms case*, Reliable Computing **13**(4) 351–359, 2007.