Mathematical Model of the Deformation and Dynamical Stability of the Abdominal Aortic Aneurysm Wall under Intravascular Pressure

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An abdominal aortic aneurysm is a local enlargement of the lower part of the aorta due to the weakening of the vessel wall. In many cases its rupture causes massive bleeding with associated high mortality. One of the main hypotheses for enlargement and rupture of the aneurysms is connected with limit point instability, that is, appearance of mathematical bifurcations in the quasi-static response of the aneurysmal wall to increased intravascular pressure. We propose a mathematical model of the deformation of the aneurysmal wall due to the inflation caused by the blood pressure. The aneurysm is supposed as a thin-walled ellipsoid which undergoes finite deformations. A system of three ordinary differential equations is obtained. In the case of isotropic aneurysmal material the system reduces to two equations. As an example, we apply the strain energy function (SEF), proposed by Raghavan and Vorp [1]. The main goal of our investigation is to analyze how blood pressure inside the aneurysm influences its dynamical stability. For that purpose we apply methods of nonlinear dynamics theory to examine local stability of the equilibrium points of the proposed system of ordinary differential equations. In addition, we apply numerical methods to elaborate our analytical results.

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References

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