Topological Analysis of Pattern-Forming Systems

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Keywords: pattern formation, topological data analysis, phyllotaxis, nanostructures

A few basic motifs, such as hexagonal lattices, are prevalent in a wide variety of pattern-forming biological and other chemical and physical systems. These include phyllotaxis (the arrangement of elements such as leaves or seeds on plants) [1], biomineralization processes [2], and nanostructures formed by the bombardment of binary materials by ions [3]. The chemical and physical mechanisms behind the formation of these similar patterns are, however, very different. It is a challenge to compare mathematical models with observations or experiments since very disparte mechanisms can produce similar steady-state patterns.

Over the last decade, a powerful new tool of computational topology called persistent homology has been developed to characterize the topological properties of a set of points or a surface at all length scales. Applying variations on this technique to the surfaces produced by simulations of pattern-forming systems, we find that, although the topological properties of the steady-state solutions are similar, the evolution of the topological properties to that steady state can differ significantly. This provides a potential way of distinguishing between mechanisms and for determining parameters.

References

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