Nonstandard Discretizations in Cancer Modeling

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Diffusion equations have been extensively studied in biosciences, contrary to cross-diffusion equations, which are more challenging from both the mathematical analysis and numerical analysis point of view. Yet, they arise naturally in cancer modeling, as seen from the book mentioned in [2]. We consider two models for cancer growth defined by cross-diffusion equations, which have positive solutions: a model for angiogenesis or classical malignant invasion [1] and a complex model for lymphangiogenesis [2]. In the two models, the underlying point is that the evolution of the tumor cell occurs in a cross-diffusive manner, which makes it difficult for nonstandard finite difference (NSFD) schemes constructed for diffusion equations to be suitable. We then design NSFD schemes that replicate the positivity of solutions by introducing a special numerical treatment of the cross-diffusion terms, combined with Mickens' rules of complex denominator function of discrete derivatives and nonlocal approximation of nonlinear terms. We provide numerical experiments that confirm the reliability of the NSFD schemes.

References

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