

# Set-Valued Approximations of Sets in $\mathbb{R}^n$

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Self-validating numerical methods are popular in practical applications since they provide results carrying within themselves a statement of their quality, namely, that the problem has a unique solution contained within a computed enclosure. The size of the enclosing set provides by itself a measure of the accuracy of approximation. The enclosures are of particular fixed type suitable for computer representation and computation, e.g. intervals (hyper rectangles), ellipsoids, zonotopes, etc. A major advantage of these methods is that uncertainties in the values of parameters can be taken into account when computing enclosures of the solution. More precisely, the range of parameter values is enclosed by a set of the type used by the vmethod and considered in the computations as a set-valued data input. The role of this advantage for applications to problems in Biomathematics is highlighted in [2]. Enclosing the parameter range by a set of predetermined type generates challenges as well. In some cases the enclosing set is substantially larger than the actual parameter range which may lead to very large enclosures of the solution with little or no information of practical significance. The difference between a set and its enclosure is the wrapping error which can be measured in different ways. An initial wrapping error can be further compounded during computations. An example is the so called wrapping effect in interval enclosure methods for systems of ODEs [1]. Here we propose a slightly different approach of *approximating* rather than *enclosing* by set of predetermined type. In this approach the quality of the approximation is measured in two different ways (i) the size of the original set not included in the enclosure and (ii) the size of the enclosure itself. We should note that (i) is similar to the error of point approximations while (ii) is similar to the error of enclosure methods. As an example, we consider the set  $\mathbb{P}$  of all regular open sets on  $\mathbb{R}^n$  approximated by a subset  $\mathbb{S}$  of  $\mathbb{P}$ . We consider on  $\mathbb{P}$  the usual Lebesgue measure and the asymmetric metric  $\rho(A, B) = \text{mes}(A \setminus B)$ . Then the error of approximation (i) of set  $A$  by a set  $M$  is measured by  $\rho(M, A)$ . It is easy to see that  $\rho(A, B) = 0 \iff A \subseteq B$ . Hence the partial order generated by  $\rho$  is exactly the set inclusion, which gives by itself a measure of the error (ii).

[1] R Anguelov, S Markov, *Wrapping Effect and Wrapping Function*, Reliable Computing **4**, 311–330 (1998)

[2] S Markov, *Biomathematics and Interval Analysis: A Prosperous Marriage*, AIP Conf. Proc. **1301**, 26 (2010); dx.doi.org/10.1063/1.3526621