

# Adjoint Sensitivity Analysis of a Spatial Tumor Growth Model with Strong Allee Effect \*

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The Allee effect is commonly used in populations models. It is said that certain population model contains Allee effect if it has a specific form of growth function. If the growth function is negative for a low population density (the model has a growth threshold whose below the population goes to extinction) then the effect is called a strong Allee effect. In case when the value of growth function is small for low population density but remains positive it is called a weak Allee effect. The Allee effects in both forms (weak and strong) is used in many models including spatial models of predator-prey interactions. Recent works [1] suggest that the strong Allee effect can be important in the spreading of malignant tumours.

In this work we use the adjoint sensitivity analysis to calculate the so-called spatiotemporal gradient of predefined objective function with respect to irradiation signal in spatial model of tumor growth [2] with introduced the strong Allee effect in the growth function. The calculated gradient can be used to more precisely predict the tumour behaviour and can be also important in developing a more patient-friendly radiotherapy protocols.

## References

- [1] L. Sewalt, K. Harley, P. van Heijster, S. Balasuriya, *Influences of Allee effects in the spreading of malignant tumours.*, J Theor Biol. **394** 77–92.
- [2] R. Rockne, E. C. Alvord, J. K. Rockhill, K. R. Swanson, *A mathematical model for brain tumor response to radiation therapy.*, J Math Biol. **58** 561–578

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