Advection Induced Patterns in the Brusselator Model

Roumen Anguelov, Stephanus Marnus Stoltz
Department of Mathematics and Applied Mathematics
University of Pretoria, South Africa
roumen.anguelov@up.ac.za, stoltzstep@gmail.com

The Brusselator model is widely used to illustrate and study basic features of models of chemical reactions involving trimolecular steps. The spatially homogeneous steady state of the model can be destabilized either via a Hopf bifurcation due to the reaction kinetics or via the Turing instability mechanism (sufficiently small ratio of the diffusion coefficients). The interplay between the two bifurcations provided an interesting subject for numerical investigation, [4], as there are no analytical results. The parameter domain comprises essentially three regions: (i) stable spatially homogeneous steady state (SS); (ii) stable Turing patterns (TP); (iii) bulk oscillations. The authors showed in [1] that the boundary of the bulk oscillations region is hyperbola-like shaped, thus confirming in more precise terms the earlier hypothesis [4] that Turing patterns eventually (for sufficiently small ratio of the diffusion coefficients) dominate the Hopf bifurcation induced oscillations. Advection induced phenomena in chemical reactions are interesting both from theoretical and practical point of view. For example, analysis of advection effects in the setting of the Schnakenberg model is presented in [2] and [3]. In this talk, which is an extension of our previous work, we consider the effect of advection on the Turing patterns in the Brusselator model as well as its role as a pattern forming mechanism, particularly for wave trains (series of waves travelling in the same direction and spaced at regular intervals). Small advection does not change qualitatively the asymptotic properties of the model. However, we observe that for advection above certain threshold value, which depends on the rest of the parameters of the model, wave trains are formed in both the TP and the SS parameter domains.

[1] R Anguelov, SM Stoltz, Stationary and oscillatory patterns in a coupled Brusselator model, MATH COMPUT SIMULAT, `dx.doi.org/10.1016/j.matcom.2015.06.002`
[2] EH Flach, S Schnell, J Norbury, Limit cycles in the presence of convection: A traveling wave analysis - Phys. Rev. E, 2007
[3] E Flach, J Norbury, S Schnell, More than Skew: Asymmetric Wave Propagation in a Reaction-Diffusion-Convection System, Biomath 2 (2013), 1303027.
[4] L. Yang, A.M. Zhabotinsky, I.R. Epstein, Stable squares and other oscillatory Turing patterns in a reactiondiffusion model, Phys. Rev. Lett., 92 (2004) 198303.