

# Multiple solutions for a nonlinear discrete fourth order $p$ -Laplacian equation

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Abstract. In this paper we study the existence of multiple solutions for a nonlinear discrete fourth order  $p$ -Laplacian equation. Similar kind of problems appears in some models from population biology. The proof of the main results is based on the three critical points theorem due to B. Ricceri [1]. An example is given.

We obtain criteria for the existence of three solutions for the following fourth order problem

$$\Delta^2 (\varphi_p (\Delta^2 u (t - 2))) + \alpha \varphi_p (u (t)) = \lambda f (t, u (t)), \quad t = 1, 2, \dots, T, \quad T \geq 2$$

with the boundary condition

$$u(0) = \Delta u(-1) = \Delta^2 u(T) = 0, \quad \Delta (\varphi_p (\Delta^2 u(T - 1))) = \mu g(u(T + 1)),$$

where  $\alpha, \lambda, \mu$  are real parameters,  $f$  and  $g$  are continuous and  $\varphi_p(x) = |x|^{p-2}x$ ,  $p \geq 2$ .

Other discrete  $p$ -Laplacian problems using variational methods are studied in [2].

## References

- [1] B. Ricceri, A further three critical points theorem, *Nonlinear Anal.* 71, 2009.
- [2] Y. Long and H. Shi, *Multiple Solutions for the Discrete  $p$ -Laplacian Boundary Value Problems*, *Discrete Dynamics in Nature and Society*, 2014.