Stability and Limit Cycles for a Predator-Prey Model with Predator Population Saturation

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A multitude of predator-prey models is discussed in literature. The models originated with Lotka (1925) and Volterra (1931). Since then there have been many refinements, but unfortunately not many of the resulting models have been tested against actual data. Thus there is no consensus even on a basic model which can be tweaked for specific prey and predators. In this paper an original predator-prey model, referred to as the Fay Greeff Hoff (FGH) model, that might just provide such a basic model, is proposed. This model subsumes many over the models found in literature and has the advantage that each term in the model has a firm ecological basis for its inclusion and form. The model is, in essence, a classical Rosenzweig-MacArthur model, but with an added function $h(y)$ that includes a constant L, called the population saturation of the predator. The FGH model therefore has bounded solutions $x(t)$ and $y(t)$ at the onset. The relevance of having bounded solutions is reflected in the Poincaré-Bendixson Theorem. Thus, the FGH model, with specific parameter values to ensure the existence of a unique equilibrium in the population quadrant, yields equilibrium values which lead to either limit cycles or attracting spiral points. From mathematical analysis a model may yield trajectories indicating the co-existence of species, when, in fact, ecologically extinction might occur. With this in mind an added constraint in the form of an extinction boundary is set. Thus the distinction is made between mathematical stability and ecological stability. In the absence of real data, the FGH model is used in an exploratory way, to gain insight in the dynamics of mathematical modelling and to explore the potential of this model as a tool in studying biological systems.