

Monotone Combined FFinite Volume-Finite Element Scheme for Anisotropic Keller-Segel Model

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The directed movement of cells and organisms in response to chemical gradients, **Chemotaxis**, has attracted significant interest due to its critical role in a wide range of biological phenomena. The Keller-Segel model proposed in [1] has provided a cornerstone for mathematical modeling of chemotaxis. We are interested by the numerical analysis of the following degenerate Keller-Segel model,

$$(1) \begin{cases} \partial_t u - \operatorname{div}(S(x)a(u)\nabla u) + \operatorname{div}(S(x)\chi(u)\nabla v) = 0 \\ \partial_t v - \operatorname{div}(M(x)\nabla v) = \alpha u - \beta v, \alpha, \beta \geq 0 \end{cases} .$$

The unknowns u and v are respectively the density of cells and the concentration of chemo-attractant. Heterogeneous and anisotropic tensors are denoted by S and M . A scheme recently developed in the finite volume framework in [2] treats the discretization of the model (1) in homogeneous domains where the diffusion tensors are considered to be the identity matrix. However, standard finite volume scheme not permit to handle anisotropic diffusion on general, possibly nonconforming meshes. In the other hand, it is well-known that finite element discretization allows a very simple discretization of full diffusion tensors and does not impose any restrictions on the meshes but many numerical instabilities may arise in the convection-dominated case. A quite intuitive idea, presented also in [3], is hence to combine a finite element discretization of the diffusion term with a finite volume discretization of the other terms. Hence, we construct and we study the convergence analysis of a combined scheme discretizing the system (1). This scheme ensures the validity of the discrete maximum principle under the classical condition that all transmissibilities coefficients are positive. Therefore, a nonlinear technique is presented, in the spirit of methods described in [4] and [5] as a correction to provide a monotone scheme for general tensors.

References

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