Pattern Formation and Cross-Diffusion for a Chemotaxis Model

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Chemotaxis is the feature movement of cell or an organism along a chemical concentration gradient. The mathematical analysis of chemotaxis models show a plenitude of spatial patterns such as the chemotaxis models applied to skin pigmentation patterns, that lead to aggregations of one type of pigment cell into a striped spatial pattern. The analysis of pattern formation can be traced to a seminal paper by Turing [1], who established that a reaction-diffusion system can generate stable nonuniform patterns in space if the components of the system interact with each other.

Our motivation is the numerical simulations of the pattern formation for a volume filling chemotaxis model. In [2], the effect of volume filling is expressed through a nonlinear squeezing probability. We investigate pattern formation using Turing's principle and the standard argument used by Murray [3,4]. Next, we introduce an implicit finite volume scheme; it is presented on a general mesh satisfying the orthogonality condition [5,6]. The originality of this scheme is the upstream approach to discretize the cross-diffusion term. Finally, we present some numerical results showing the spatial patterns for the chemotaxis model.

References

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