Numerical Analysis of the Coupled Modified Van der Pol Equations in a Model of Heart Action

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In this paper, modified van der Pol equations are considered as description of the heart action. Wide ranges of the model parameters yield interesting qualitative results, e.g. Hopf bifurcation, Bogdanov-Takens bifurcation, transcritical and pitchfork bifurcations, but also stable solutions can be found. The physiological model works in nearest range of parameters and allows to obtain stable behaviour which is important for solving the biological problem. When some kinds of pathologies appear in the heart, it is possible to obtain a chaotic behaviour. My aim is to compare the influence of two types of coupling (unidirectional and bidirectional) on the behaviour of the van der Pols system. The coupling takes place in the healthy conductivity system between two nodes, SA and AV, but in some circumstances the pathological coupling can occur in the heart. The Van der Pol oscillator is a type of a relaxation oscillator, which can be synchronized. Synchronization properties of such a system is studied in the following work. For numerical analysis of the discussed system a numerical model was created.

References

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